On Topological Contra θgs-Quotient Functions

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Abstract— The aim of this paper is to introduce contra θgs-Quotient function using θgs-closed sets and study their basic properties. We study the relation between weak and strong form of contra θgs-Quotient functions. We also derive relation between strongly θgs-Continuous function and Contra θgs-Quotient function.

Keywords: θgs-open set, θgs-closed set, θgs-Quotient function,

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I. Introduction

In 1970, Levine [4] offered a new and useful notion called Generalized closed set in General Topology. This notion has been studied extensively in recent years by many topologists. The investigation of generalized closed sets had led to several new and interesting concepts. After the introduction of generalized closed sets there are many research papers which deal with different types of generalized closed sets. Navalagi and Md.Hanif Page [5] have introduced the notion of θ-generalized semi closed (briefly, θgs-closed) sets and studied their properties. The aim of this paper is to introduce contra θgs-quotient functions using θgs-closed sets and using these new types of functions, several characterizations and its properties have been obtained

II. PRELIMANARIES

Throughout this paper (X,τ) and (Y,σ) (or simply X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. If A is any subset of space X, then Cl(A) and Int(A) denote the closure of A and the interior of A in X respectively.

The following definitions are useful in the sequel:

Definition 2.1: A subset A of space X is called
(i) a semi-open set [3] if $A \subseteq Cl(Int(A))$
(ii) a semi-closed set [1] if $Int(Cl(A)) \subseteq A$

Definition 2.2 [2]**:** A point $x \in X$ is called a semi-θ-cluster point of A if $A \cap sCl(U) \neq \varphi$ for each semi-open set U containing x.
The set of all semi-θ-cluster point of A is called semi- θ-closure of A and is denoted by $sCl_\theta(A)$. A subset A is called semi-θ-closed if $sCl_\theta(A) = A$. The complement of semi-θ-closed set is semi-θ-open set.

Definition 2.3 [5]**:** A subset A of a topological space X is called θ-generalized-semi closed (briefly, θgs-closed) if $sCl_\theta(A) \subset U$, whenever $A \subset U$ and U is open in X. The complement of θgs-closed set is θ-generalized-semi open (briefly, θgs-open). We denote the family of θgs-closed sets of X by $\theta GSC(X, \tau)$ and θgs-open sets by $\theta GSO(X, \tau)$.

Definition 2.4 [8]: A topological space X is called $T_{\theta gs}$ -space if every θgs-closed set in it is closed set.

Definition 2.5: A function $f: X \to Y$ is called
(i) θgeneralized semi-continuous (in briefly, θgs-continuous) [6], if $f^{-1}(F)$ is θgs-closed in X for every closed set F of Y.
(ii) θ-generalized semi-irresolute (in briefly, θ-gs-irresolute) [6], if $f^{-1}(F)$ is θgs-closed in X for every θgs-closed set F of Y.
(iii)Strongly θ-generalized semi-continuous (briefly, strongly θgs-continuous) [10] if $f^{-1}(F)$ is closed set of X for each θgs-closed set F of Y.
(iv) Contra θgs-continuous [11] if $f^{-1}(F)$ is θgs-closed set in X for each open set F of Y.

Definition 2.6 [7]: A function $f: X \to Y$ is θgs-open (resp., θgs-closed) in Y for every open set (resp., closed) V in X.

Definition 2.7 [12]: A function $f: X \to Y$ is said to be
(i) θgs-quotient if f is θgs-continuous and $f^{-1}(V)$ is open in X implies V is θgs-open in Y
(ii) Strongly θgs-quotient if f is θgs-continuous and $f^{-1}(V)$ is open in X implies V is θgs-open in Y.
(iii) Strongly θgs-open if f(U) is θgs-open in Y for each θgs-open set U in X.

III. CONTRA θGS-QUOTIENT FUNCTIONS

Definition 3.1 : A surjective function $f: X \to Y$ is said to be contra-θ-generalized semi- quotient (briefly, Contra θgs-quotient) if f is contra θgs-continuous and $f^{-1}(V)$ is closed in X implies V is θgs-open in Y .

Definition 3.2: A surjective function $f: X \to Y$ is said to be contra-strongly θgs- quotient provided a set V of Y is open in Y if and only if $f^{-1}(V)$ is θgs-closed in X

Definition 3.3: A surjective function $f: X \to Y$ is said to be contra-strongly θgs-closed if the image of every θgs-closed in X is θgs-open in Y.

Theorem3.4: Every contra-strongly θgs-quotient function is contra-θgs-quotient function but converse is not true.
Proof: Let f: X $\rightarrow$ Y be contra-strongly θgs-quotient function. Then $f^{-1}(V)$ θgs-closed in X. This implies that if f is contra θgs-continuous and surjective. Let f^{-1} (V) closed in X. Then $f^{-1}(V)$ θgs-closed in X. By hypothesis, V is open in Y. Then V is θgs-open in Y. Therefore f is θgs-quotient function.

Example 3.5: Let X=Y= {a, b, c}, τ ={X, φ, {a},{b}, {a, c}} and σ = {Y, φ, {a}, {b},{a, b}} be topologies on X and Y respectively. We have θGSO(X) ={X, φ, {b}, {a, c},{b, c}} and θGSO(Y) ={Y, φ ,{a},{b, c}}. Define a function f: X $\rightarrow$ Y by f(a) = b, f(b) = a, f(c) = c. Then f is contra θgs-quotient function but not contra strongly θgs-quotient function, because for an open set {a} in Y, f^{-1} ({a}) ={b} is not θgs-closed in X

Remark 3.6: θgs-quotient functions and contra θgs-quotient functions are independent of each other as shown below.

Example3.7: Let X=Y= {a, b, c}, τ ={X, φ, {b},{c}, {a, b}, {a, c}} and σ = {Y, φ, {a}, {b}, {a, b}} be topologies on X and Y respectively. We have θGSO(X) ={X, φ , {b}, {c}, {a, b},{b ,c}} and θGSO(Y) ={Y, φ ,{a},{b, c}}. Define a function f: X $\rightarrow$ Y by f(a) = b, f(b) = c, f(c) = a. Then f is contra θgs-quotient function but not θgs-quotient function, since f is not θgs-continuous function.

Example3.8: Let X=Y= {a, b, c}, τ ={X, φ, {a},{b}, {a, b}} and σ = {Y, φ, {a},{b},{a, c}} be topologies on X and Y respectively. We have θGSO(X) ={X, φ, {a},{b,c}} and θGSO(Y) ={Y, φ ,{b},{c},{b, c}}. Define f: X $\rightarrow$ Y by f(a) = a, f(b) = b, f(c) = c. Then f is θgs- quotient function but not contra θgs-quotient function, because for an θgs -open set {b} in Y, f^{-1} ({b}) = {b} is not closed set in X

Remark 3.9: The concepts of contra θgs-closed functions and contra θgs-quotient functions are independent of each other as shown below.

Example3.10: In Example 3.7, f is contra θgs-quotient but not contra strongly θgs-closed, as f ({a, c}) = {a, b} which is not θgs-open set in Y.

Theorem 3.12: Every contra-strongly θgs-quotient function is contra-strongly θgs-closed but converse is not true.
Proof: Let f: X $\rightarrow$ Y be contra-strongly θgs-quotient function. Let V be a θgs-closed set in X. That is $f^{-1}(f(V))$ is θgs-closed in X. By hypothesis, f (V) is open in Y. Hence f is contra-strongly θgs-
closed function.

Example 3.13: Let X=Y= {a, b, c}, τ ={X, φ, {b}, {c}, {a,b},{a, c}} and σ = {Y, φ , {b},{c},{b, c}} be topologies on X and Y respectively. We have θGSC(X) ={X, φ , {a},{ c},{a,b},{a,c}}and θGSO(Y) ={Y, φ,{b},{c},{b,c},{a,c} {a,b}}. Define a function f: X $\rightarrow$ Y by f(a) = c, f(b) = a, f(c) = b. Then f is contra-strongly θgs-closed function but not contra strongly θgs-quotient function, because f^{-1}({c}) = {a} is θgs-closed in X but not an open set in Y.

Remark 3.14: The following example shows that contra-strongly θgs-quotient functions and strongly θgs-closed functions are independent of each other.

Example3.15: Let X=Y= {a, b, c}, τ ={X, φ, {b}, {c}, {a,b},{a, c}} and σ = {Y, φ, {b},{c},{ b, c}} be topologies on X and Y respectively. We have θGSO(X) ={X, φ , {b},{ c},{a, b},{b, c}} and θGSO(Y) ={Y, φ ,{b},{c},{b, c},{a, c},{a, b}}. Define a function f: X $\rightarrow$ Y by f(a) = a, f(b) = b, f(c) = c. Then f is strongly θgs-closed function but not contra strongly θgs-quotient function, because for an open set {b}in Y, f^{-1}({b}) ={b} is not θgs-closed set in X.

Example3.16: Let X=Y= {a, b, c}, τ ={X, φ , {b}, {c}, {a,b},{a, c}} and σ = {Y, φ, {a},{b},{a, b}} be topologies on X and Y respectively. We have θGSO(X) ={X, φ , {b},{ c},{a, b},{b, c}} and θGSO(Y) ={Y, φ ,{a}{b, c}}. Define a function f: X $\rightarrow$ Y by f(a) = a, f(b) = c, f(c) = b. Then f is contra-strongly θgs-quotient function but not contra strongly θgs-closed function, because for θgs-closed set {a, c} in X, f^{-1}({a, c}) = {a, b} is not θgs-closed in X.

Remark3.17: Contra-strongly θgs-quotient functions and strongly θgs-quotient functions are independent of each other as shown below.

Example 3.18: In Example 3.16, the function f is contra strongly θgs-quotient function but not strongly θgs-quotient function because f is not θgs-continuous function.

Example3.19: Let X=Y= {a, b, c}, τ ={X, φ, {b}, {c}, {b, c}} and σ = {Y, φ, {a, b},{a, c},{b},{c}} be topologies on X and Y respectively. We have θGSO(X) ={X, φ, {b}, {c}, {a, b}, {b, c}, {a, c}} and θGSO(Y) ={Y, φ, {b}, {b, c}, {a, b}. Define a function f: X $\rightarrow$ Y by f(a) = a, f(b) =b, f(c) =c. Then f is strongly θgs-quotient function but not contra strongly θgs-quotient function, because f^{-1}({a}) = {a} is θgs-closed in X. but not an open set in Y.

Definition 3.20: A function f: X $\rightarrow$ Y is said to be contra-θgs-irresolute if the inverse image of every θgs-open set in Y is θgs-closed in X.

Definition 3.21: A function f: X $\rightarrow$ Y is said to be contra-completely-θgs-quotient if f is surjective, contra-θgs-irresolute and $f^{-1}(V)$ is θgs-closed in X implies V is open in Y.

Theorem3.22: Every contra-completely- θgs-quotient function is contra- θgs-irresolute.
Proof: Follows from the definitions.

Theorem 3.23: A contra- θgs-irresolute function need not be a contra-completely- θgs-quotient function as shown in the following example.

Example3.24: Let X=Y= {a, b, c}, τ ={X, φ , {b}, {c}, {b, c}} and σ = {Y, φ, {a, b},{a, c},{b},{c}} be topologies on X and Y respectively. We have θGSO(X) ={X, φ , {b}, {c},{a, b},{b, c},{a, c}}and θGSO(Y) ={Y, φ,{b},{b, c},{a, b}. Define a function f: X $\to$ Y by f(a) = a, f(b) =b, f(c) = c. Then f is contra θgs-irresolute function but not contra-completely-θgs-quotient function, because f^{-1}({a}) = {a} is θgs-closed in X but {a} is not an open set in Y.

Theorem 3.25: Every contra-completely- θgs-quotient function is contra-θgs-quotient.

Proof: Let f: X $\to$ Y be contra-completely-θgs-quotient function. Let V be an open set in Y. Then V is θgs-open in Y as f is contra-θgs-irresolute, f^{-1}(f (V)) is θgs-closed in X. This implies that f is contra- θgs-irresolute. Clearly f is surjective. Let f^{-1}(V) be a closed in X where V $\subseteq$ Y. Then f^{-1}(V) is a θgs-closed in X. By, hypothesis, V is open in Y and hence θgs-open. Hence f is contra- θgs-quotient function.

Remark 3.26: The converse of above theorem need not be true as shown in the following example.

Example3.27: Let X=Y= {a, b, c}, τ ={X, φ, {a}, {b}, {c},{a, b},{a, c}} and σ = {Y, φ ,{a},{b},{a, b}} be topologies on X and Y respectively. We have θGSO(X) ={X, φ,{a},{b},{c},{a, b},{a, c},{b, c}}and θGSO(Y) ={Y, φ ,{a},{b, c}}. Define a function f: X $\to$ Y by f(a) =b, f(b) = a, f(c) = c. Then f is contra θgs-quotient function but not contra-completely-θgs-quotient function, because f^{-1}({b, c}) = {a, c} is θgs-closed in X but {a, c} is not an open set in Y.

III. APPLICATIONS

Theorem 4.1: Let f: X $\to$Y is a closed, surjective and θgs-irresolute and g: Y $\to$Z be a contra-θgs-quotient function, then gof: X $\to$ Z is contra-θgs-quotient function.

Proof: Let V be a closed set in Z. Since g is contra- θgs continuous, g^{-1}(V) is a θgs-open in Y. Since f is θgs irresolute $f^{-1}(g^{-1}(V))$ = $(gof)^{-1}$(V) is θgs-open in X. Therefore (gof) is contra- θgs-continuous. Assume that $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is closed in X for some subset V in Z. Since f is closed, implies $f((gof)^{-1}(V))$ is closed in Y. Since f is surjective, g^{-1}(V) is closed in Y. Since g is contra- θgs-quotient function, implies V is a θgs-open set in Z. This shows that, (gof) is contra-θgs-quotient function.

Theorem 4.2: Let f: X $\to$ Y is surjective, strongly θgs-closed and θgs-irresolute and g:Y$\to$Z is contra-completely-θgs-quotient function, then gof: X $\to$Z is contra-completely-θgs-quotient function.

Proof: Let V be an open set in Z. Since g is contra-completely-θgs-quotient function then ,g^{-1}(V) is a θgs-closed set in Y. Since f is θgs-irresolute $f^{-1}(g^{-1}(V))$ = $(gof)^{-1}$(V) is θgs-closed set in X. Hence (gof) is contra θgs irresolute. Let $(gof)^{-1}$(V)) is θgs-closed set in X. Since f is strongly θgs-closed function, $f(f^{-1}(g^{-1}(V))$ is θgs-closed set in Y. That is g^{-1}(V) is θgs-closed set in Y. Since g is contra-completely-θgs-quotient function, implies V is an open set in Z. Thus (gof) is contra-completely- θgs-quotient function.

Theorem 4.3 : If f : X $\to$ Y and g : Y $\to$ Z be any two functions such that g o f :X $\to$ Z

(i) If (gof) is strongly θgs-closed and g is contra-θgs-irresolute injective then f is contra- strongly θgs-closed.

(ii) If (gof) is contra-θgs-irresolute and g is contra-strongly θgs-closed injective then f is

Proof: (i) Let V be a θgs-closed set in X. Then (gof)(V) is a θgs-closed in Z. Since g is contra θgs-irresolute, g^{-1}((g o f)(V)) is θgs-open in Y .That is f(V) is θgs-open in Y. Hence f is contra-strongly θgs-closed.

(ii) Let V be a θgs-closed in Y. Since g is contra-strongly θgs-closed, g(V) is θgs-open in Z. Since (g $\circ$ f) is contra θgs-irresolute, $(g o f)^{-1}$ (g (V))) θgs-closed in X. That is g^{-1}(V) is θgs-closed set in X. Hence f is θgs-irresolute.

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